Journal of Engineering Physics and Thermophysics, Vol. 74, No. 1, 2001

COMPLEX MODEL OF THE EFFICIENCY OF RECTIFICATION PLATES.
2. COUNTERCURRENT MOTION OF THE PHASES

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UDC 66.048.375

Consideration is given to the mass exchange in countercurrent motion and ideal displacement of the phases on a plate when the vapor and incoming liquid that leave an ideal plate are in the equilibrium state. In the complex model of mass exchange proposed, the compositions of the flows on the ideal and actual plates coincide at a certain distance from the site of injection of the phases. Equality of the efficiencies in the vapor and liquid phases in countercurrent motion of them is proved. Certain particular and boundary cases of the complex model are considered. Corresponding relations of the efficiencies are derived. The countercurrent complex model and the fields of its application are analyzed.

In the Murphree [1] and Hausen [2] models, which have received the widest acceptance in the analysis of the efficiency of rectification columns, one condition is the equilibrium of the vapor and liquid that leave the plate. This condition is valid in total mixing of the liquid on the plate and in concurrent motion of the phases. In ideal displacement and countercurrent motion of the phases the liquid that arrives at the plate is in equilibrium with the vapor leaving (Fig. 1). Use of the Murphree and Hausen models in this case leads to significant errors since in them the concentrations of the highly volatile component in the outgoing vapor are underestimated and the values of the mass-exchange efficiency are accordingly distorted. Therefore the Murphree and Hausen models cannot cover the entire variety of implementation of rectification processes. This is confirmed by literature data since, according to some researchers, it is the Murphree model [1, 3, 4] that has satisfactory convergence with experimental values, according to others it is the Hausen model [2, 5–7], and according to still others satisfactory convergence is displayed by other mass-exchange models [8-10].

In [11], the author points to certain drawbacks of the existing models that are caused by the conditions of the relationship between the ideal and real plates and by the equilibrium conditions. Similar drawbacks also occur in the case of countercurrent motion of the interacting phases, where the actual plate is somewhat idealized and its efficiency is overestimated.

In particular, for the Murphree models [1, 5, 6] when the efficiency in the vapor and liquid phases is analyzed and for the Hausen model [2, 5, 6] the conditions of the relationship of the ideal and real plates can be represented in the form of equalities (2) that are given in [11].

As applied to countercurrent motion of the phases, the conditions of equilibrium of the phases on the ideal plate are formulated as follows (Fig. 1):

$$y_n^* = mx_n^* . \tag{1}$$

Equalities (2a) of [11] have no effect on dependence (1); the latter is in agreement with the idea of the operation of the ideal plate, and the reliability of the model is governed by only the conditions of the relationship of the ideal and real plates.

Belarusian State Technological University, Minsk, Belarus. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 74, No. 1, pp. 57–61, January–February, 2001. Original article submitted January 26, 2000; revision submitted May 31, 2000.

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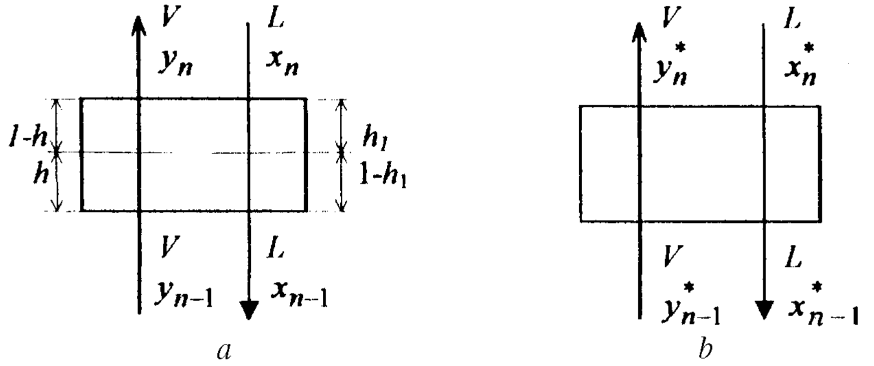

Fig. 1. Scheme of countercurrent flows of the vapor and liquid phases on the real (a) and ideal (b) plates.

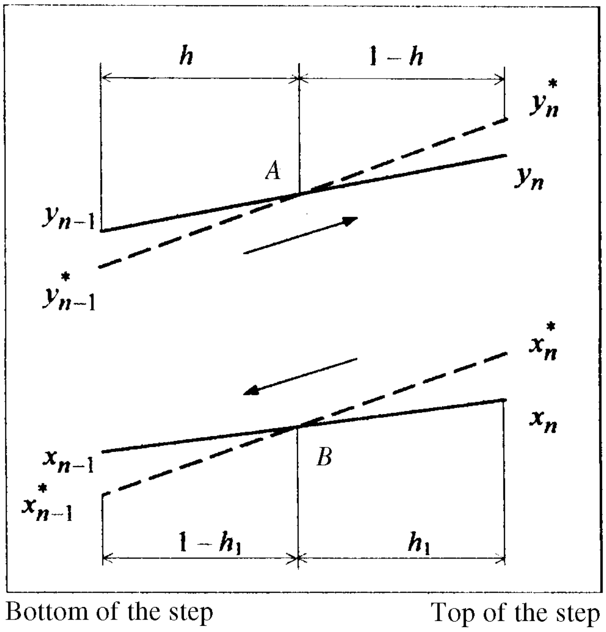

Fig. 2. Change in the concentrations on the real (solid lines) and ideal (dashed lines) plates in countercurrent phase motion.

With account for equality (2b) of [11] formula (1) acquires the form $y_n = mx_n$, according to which the concentrations of the highly volatile component in the vapor phase after the real plate and in the liquid before it are overestimated while the corresponding quantities for the ideal plate are underestimated, thus distorting the operating characteristics of the two plates.

Employment of equalities (2c) of [11] in the equilibrium condition (1) leads to the dependence $y_n^* = mx_n$, using which we obtain overestimated concentrations of the highly volatile component in the liquid that arrives at the actual plate and underestimated concentrations in the vapor phase that leaves the ideal plate. The parameters of the models of the ideal and real plates are also distorted.

Thus, in countercurrent phase motion the calculated values can differ significantly from experimental data since this difference is built-in in the formation of the models.

By analogy with concurrent motion [11], in countercurrent motion of the liquid and the vapor we propose the following mass-exchange model (Fig. 2). The incoming and outgoing flows of the vapor and the liquid on the ideal and real plates differ in composition and are equalized at a certain distance from the site of injection of the phases h for the vapor and h_1 for the liquid (points A and B in Fig. 2, respectively). In this model, the equations of material balance of the highly volatile component for the ideal and real plates and the expressions for the efficiency of the mass exchange in the vapor and liquid phases have the same form as in concurrent motion [11]. Therefore in countercurrent motion we can similarly obtain a relation for the concentrations of the highly volatile component in the vapor phase and the liquid on the ideal and real steps and concentrations of the highly volatile component in the vapor phase before and after the ideal plate that are identical to those in concurrent motion and are expressed by the parameters of the real plate [11].

Countercurrent motion differs from concurrent only in the equilibrium equation (1). Taking this into account, we can obtain, using the efficiency of the mass transfer in the vapor phase, the difference in the concentrations of the highly volatile component in the liquid on the ideal and real steps, respectively:

$$x_n^* - x_{n-1}^* = \frac{h}{h_1}\frac{y_n}{m} + \frac{1-h}{h_1}\frac{y_{n-1}}{m} + \frac{1-h}{h_1}\frac{y_n - y_{n-1}}{mE_{\mathrm{v}}} - \frac{1-h_1}{h_1} x_n - x_{n-1}\,, \tag{2}$$

$$x_n - x_{n-1} = \frac{\left(x_{n-1} - \dfrac{y_{n-1}}{m}\right)E_{\mathrm{v}}}{L\,(1-E_{\mathrm{v}})\left(\dfrac{1-h}{mV} + \dfrac{1-h_1}{L}\right) + \dfrac{L}{mV}E_{\mathrm{v}} - 1}\,, \tag{3}$$

and using the efficiency in the liquid we can obtain the difference in the concentrations of the highly volatile component in the vapor phase on the ideal plate

$$y_n^* - y_{n-1}^* = \frac{1-h_1}{1-h} m x_n + \frac{h_1}{1-h} m x_{n-1} + \frac{h_1}{1-h} m \frac{x_n - x_{n-1}}{E_{\mathrm{liq}}} - \frac{h}{1-h} y_n - y_{n-1} \tag{4}$$

and the difference

$$x_n - x_{n-1} = \frac{\left(x_{n-1} - \dfrac{y_{n-1}}{m}\right)E_{\mathrm{liq}}}{L\,(1-E_{\mathrm{liq}})\left(\dfrac{1-h}{mV} + \dfrac{1-h_1}{L}\right) + \dfrac{L}{mV}E_{\mathrm{liq}} - 1}\,.$$

A comparative analysis of the last expression and (3) confirms the equality of the efficiencies in the vapor and liquid phases for the mass-exchange model in question, i.e.,

$$E_{\mathrm{v}} = E_{\mathrm{liq}} = E_{\mathrm{g}}\,. \tag{5}$$

In the particular case where the distance h can be equal to h_1 formula (3) is simplified:

$$x_n - x_{n-1} = \frac{\left(x_{n-1} - \dfrac{y_{n-1}}{m}\right)E_{\mathrm{g}}}{(1-E_{\mathrm{g}})\,(1-h)\left(\dfrac{L}{mV} + 1\right) + \dfrac{L}{mV}E_{\mathrm{g}} - 1}\,, \tag{6}$$

while for $h = h_1 = 0.5$ it is reduced to the form

$$x_n - x_{n-1} = \frac{\left(x_{n-1} - \frac{y_{n-1}}{m}\right) 2E_{\mathrm{g,m}}}{\left(\frac{L}{mV} - 1\right)(1 + E_{\mathrm{g,m}})} . \tag{7}$$

The boundary conditions of the complex model are the conditions of the relationship of the ideal and real plates that are inherent in the Murphree [1, 5, 6] and Hausen [2, 5, 6] models when the distances h and h_1 become equal to zero or unity. For $y^*_{n-1} = y_{n-1}$ and $x^*_{n-1} = x_{n-1}$ these quantities are specified as $h = 0$ and $h_1 = 1$. In this case, dependence (3) is simplified:

$$x_n - x_{n-1} = \frac{\left(x_{n-1} - \frac{y_{n-1}}{m}\right) E_{\mathrm{g}1}}{\frac{L}{mV} - 1} . \tag{8}$$

If $y^*_n = y_n$ and $x^*_n = x_n$, which, as applied to the model in question, corresponds to $h = 1$ and $h_1 = 0$, expression (3) acquires the form of formula (8) in which the efficiency is equal to unity:

$$x_n - x_{n-1} = \frac{x_{n-1} - \frac{y_{n-1}}{m}}{\frac{L}{mV} - 1} . \tag{9}$$

With allowance for this fact it should be noted that it seems impossible to use expression (9) for calculating the parameters of the actual plate in countercurrent motion of the phases since the mass-exchange efficiency must be less than unity.

When $y^*_{n-1} = y_{n-1}$ and $x^*_n = x_n$ the quantities h and h_1 take on zero values, and dependence (3) is simplified:

$$x_n - x_{n-1} = \frac{\left(x_{n-1} - \frac{y_{n-1}}{m}\right) E_{\mathrm{g}3}}{\frac{L}{mV} - E_{\mathrm{g}3}} . \tag{10}$$

If $y^*_n = y_n$ and $x^*_{n-1} = x_{n-1}$ then $h = 1$ and $h_1 = 1$, and dependence (3) acquires the form

$$x_n - x_{n-1} = \frac{\left(x_{n-1} - \frac{y_{n-1}}{m}\right) E_{\mathrm{g}4}}{\frac{L}{mV} E_{\mathrm{g}4} - 1} . \tag{11}$$

The left-hand sides in formulas (7), (8), (10), and (11) are equal. By equating their right-hand sides we obtain relations between the efficiencies:

$$\frac{\frac{L}{mV} - 1}{2E_{\mathrm{g,m}}} + \frac{L}{2mV} - \frac{1}{2} = \frac{\frac{L}{mV} - 1}{E_{\mathrm{g}1}} = \frac{\frac{L}{mV}}{E_{\mathrm{g}3}} - 1 = \frac{L}{mV} - \frac{1}{E_{\mathrm{g}4}} . \tag{12}$$

Dependences for determining the efficiency of the real plate based on experimental data can be derived from the generalized formula (3) with account for (5):

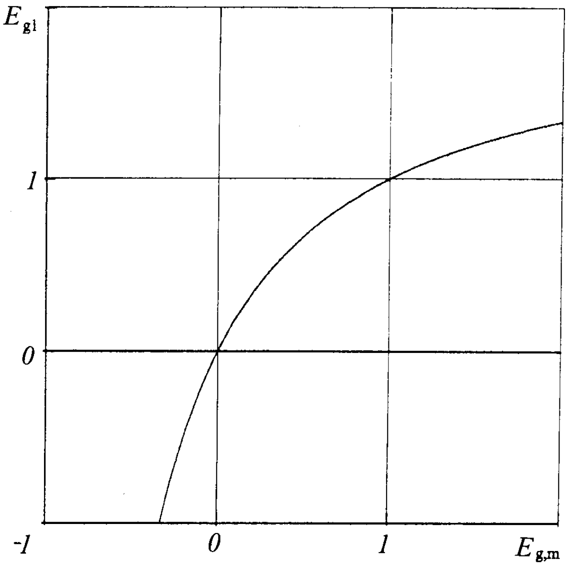

Fig. 3. Dependence of E_{g1} on $E_{g,m}$.

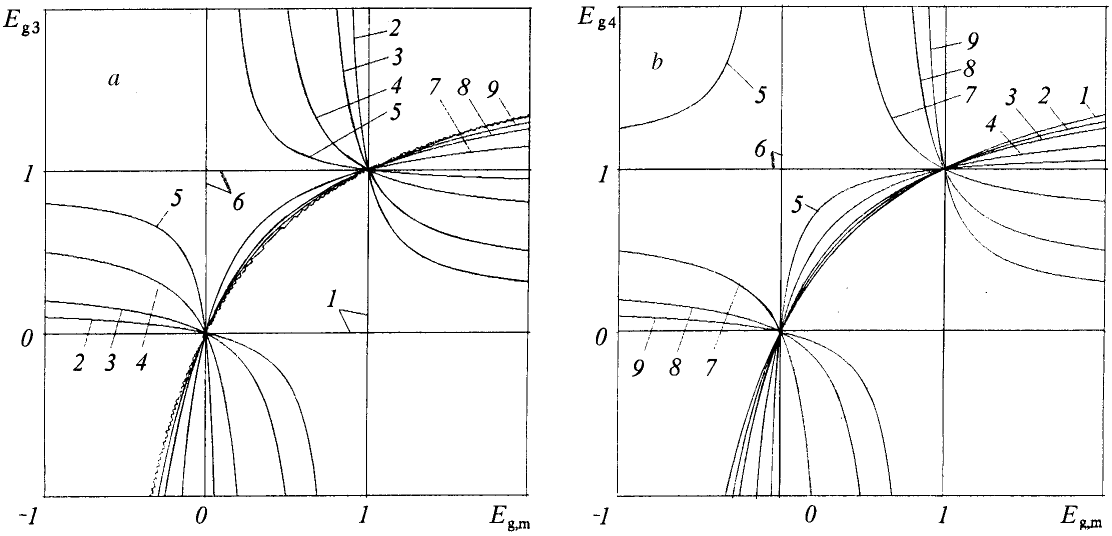

Fig. 4. Dependence of E_{g3} (a) and E_{g4} (b) on $E_{g,m}$ for different values of L/mV: 1) 0; 2) 0.1; 3) 0.2; 4) 0.5; 5) 0.8; 6) 1.0; 7) 2.0; 8) 5.0; 9) 10.

$$E_g = \frac{(1-h)\dfrac{L}{mV} - h_1}{\dfrac{x_n - \dfrac{y_{n-1}}{m}}{x_n - x_{n-1}} - h\dfrac{L}{mV} - h_1} \tag{13}$$

or from its modification (7) for $h = h_1 = 0.5$:

$$E_{g,m} = \frac{\dfrac{L}{mV} - 1}{\dfrac{x_n + x_{n-1} - 2\dfrac{y_{n-1}}{m}}{x_n - x_{n-1}} - \dfrac{L}{mV}}\,. \tag{14}$$

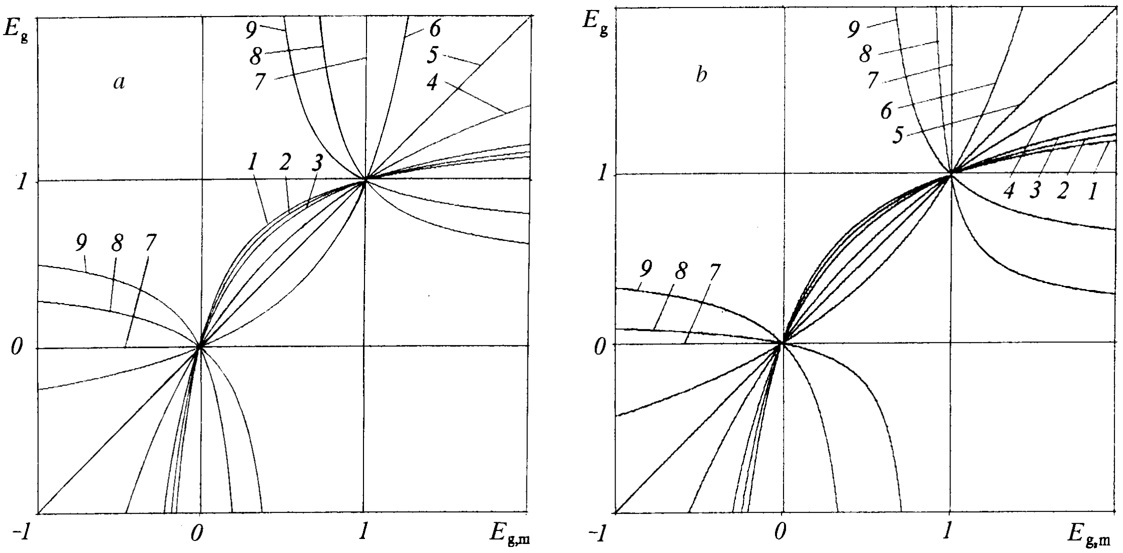

Fig. 5. Dependence of E_g on $E_{g,m}$ for $L/mV = 1.5$, $h = 0.5$ (a), $h_1 = 0.5$ (b), and different values of h_1 (a) and h (b): 1) 0; 2) 0.1; 3) 0.2; 4) 0.4; 5) 0.5; 6) 0.6; 7) 0.75 (a) and 0.666 (b); 8) 0.8; 9) 1.0.

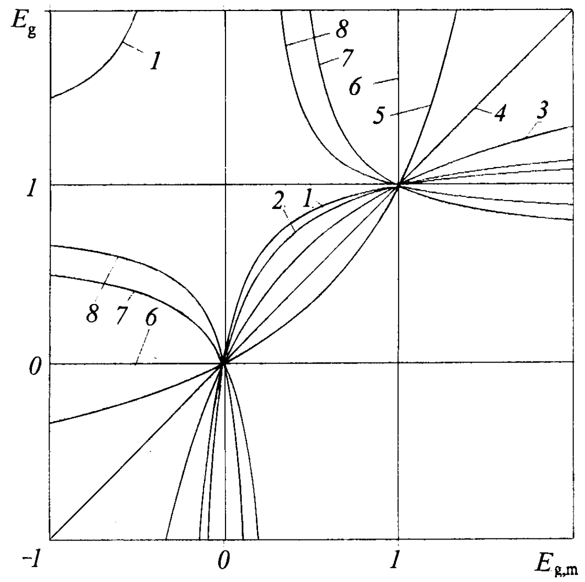

Fig. 6. Dependence of E_g on $E_{g,m}$ for $L/mV = 1.5$, $h_1 = h$, and different values of h: 1) 0; 2) 0.2; 3) 0.4; 4) 0.5; 5) 0.55; 6) 0.6; 7) 0.8; 8) 1.0.

Figures 3 and 4 give graphical representations of the dependences of individual pairs of efficiencies of the complex model of mass exchange in countercurrent phase motion in accordance with formula (12).

As is seen from Fig. 3, the values of E_{g1} that correspond to countercurrent mass exchange and conditions of the relationship between the ideal and real plates that are similar to the Murphree model in analyzing the efficiency in the vapor phase are higher than the values of $E_{g,m}$ that correspond to the complex model in the real range of efficiencies ($0 < E < 1$) for any values of the parameter L/mV. This confirms the assumption that the real plate in which $y_{n-1} = y^*_{n-1}$ and $x_{n-1} = x^*_{n-1}$ is somewhat idealized and that the actual conditions of the mass exchange on the plate correspond more fully to the complex model.

The efficiency E_{g3} that meets the requirements of the conditions of the relationship between the real and ideal plates in the Hausen model but in countercurrent phase motion is higher than $E_{g,m}$ in the real range for $L/mV > 1$ (Fig. 4a). With increase in L/mV the difference in the efficiencies decreases but does not disappear completely, which also confirms the fuller correspondence of the complex model to the actual conditions. For $L/mV < 1$, the real values of one efficiency correspond to nonreal values of the other in any range of their values.

A similar relationship occurs between the efficiencies E_{g4} and $E_{g,m}$ (Fig. 4b). Unlike the preceding figure, $E_{g4} > E_{g,m}$ for $L/mV < 1$, and the nonreal values of one efficiency correspond to nonreal values of the other for $L/mV > 1$.

The analysis performed shows that, in the complex model, use of the boundary conditions of the relationship of the ideal and real plates when h or h_1 is equal to zero or unity leads to overestimation of the efficiency as compared to its value for $h = h_1 = 0.5$, i.e., the actual plate is slightly idealized by the assignment of the boundary conditions, and certain intermediate values of the efficiencies can be real. This is also confirmed by Figs. 5 and 6.

From Fig. 5a it follows that for the assigned values $L/mV = 1.5$ and $h = 0.5$ a decrease in h_1 from the average value improves the efficiency while an increase above 3/4 of the total value of the distance leads to nonreal values. This points to the possibility of using the complex model for $0.5 < h_1 < 0.75$ but the region of $h > 0.5$ is preferable.

For fixed values of L/mV and h_1 (Fig. 5b) the efficiency increases with decrease in h and takes on nonreal values for $h > 0.666$. Under these conditions, it is appropriate to use the complex model in the range of $0.5 < h < 0.666$.

A simultaneous increase in the distances h and h_1 (Fig. 6) decreases the efficiency of the mass exchange as compared to their average values; $E_g < E_{g,m}$ if $h = h_1 > 0.5$ and $E_g > E_{g,m}$ if these distances are less than half. As is seen from Figs. 5 and 6, under certain conditions, $E_g < E_{g,m}$; this is probably due to the fact that in the complex model the efficiency is minimum when h and h_1 differ not only from the boundary values but also from the average value.

The complex model of mass exchange proposed extends the field of application and diminishes negative circumstances caused by both the conditions of formation of the existing models and the character of the phase motion.

NOTATION

E, efficiency of the plate; h and h_1, dimensionless distance from the site of injection of the vapor and the liquid, respectively, to the surface of equality of the concentrations of the phases in the ideal and actual plates; L, molar flow of the liquid; m, coefficient of equilibrium; V, molar flow of the vapor; x and y, concentration of the highly volatile component in, respectively, the liquid and the vapor. Subscripts and superscripts: g, countercurrent motion of the phases; g,m countercurrent motion of the phases for $h = h_1 = 0.5$; liq, liquid phase; n, number of the plate in question; n–1, number of the preceding plate in the direction of vapor motion; v, vapor phase; *, ideal conditions.

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